Phylogenetic Reconstruction: Distance Matrix Methods

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Trees: terminology

- Terminal node (leaf)
- Internal node (hypothetical ancestor)
- Branch (edge)
- Root
Trees: terminology

- **Star tree**
- **Partially resolved**
- **Fully resolved**

**Polytomy**

- **Monophyletic**
- **Non-monophyletic**
Trees: representations

Three different representations of the same tree
Trees: representation in computer files

Newick format:

• Leaf nodes: represented by taxon name
• Internal nodes: represented by pair of matching parentheses
• Descendants of internal node given as comma-delimited list.
• Tree string terminated by semicolon

((A, B), (C, D));
Newick format: named for seafood restaurant where standard was decided upon
Trees: representation in computer files

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Trees: rooted vs. unrooted

- A rooted tree has a single node (the root) that represents a point in time that is earlier than any other node in the tree.
- A rooted tree has directionality (nodes can be ordered in terms of “earlier” or “later”).
- In the rooted tree, distance between two nodes is represented along the time-axis only (the second axis just helps spread out the leafs).
Trees: rooted vs. unrooted

- In unrooted trees there is no directionality: we do not know if a node is earlier or later than another node

- Distance along branches directly represents node distance
Reconstructing a tree using non-contemporaneous data
Reconstructing a tree using present-day data
Data: molecular phylogeny

- DNA sequences
  - genomic DNA
  - mitochondrial DNA
  - chloroplast DNA

- Protein sequences

- Restriction site polymorphisms

- DNA/DNA hybridization

- Immunological cross-reaction
Morphology vs. molecular data

New and old world vultures seem to be closely related based on morphology. Molecular data indicates that old world vultures are related to birds of prey (falcons, hawks, etc.) while new world vultures are more closely related to storks. Similar features presumably the result of convergent evolution.
Molecular data: single-celled organisms

Molecular data useful for analyzing single-celled organisms (which have only few prominent morphological features).
Distance Matrix Methods

1. Construct multiple alignment of sequences

<table>
<thead>
<tr>
<th></th>
<th>Go</th>
<th>Hu</th>
<th>Ch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go</td>
<td></td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Hu</td>
<td>-</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Ch</td>
<td></td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

2. Construct table listing all pairwise differences (distance matrix)

3. Construct tree from pairwise distances
Finding Optimal Branch Lengths

<table>
<thead>
<tr>
<th></th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
<th>S₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>-</td>
<td>D₁₂</td>
<td>D₁₃</td>
<td>D₁₄</td>
</tr>
<tr>
<td>S₂</td>
<td></td>
<td>-</td>
<td>D₂₃</td>
<td>D₂₄</td>
</tr>
<tr>
<td>S₃</td>
<td></td>
<td></td>
<td>-</td>
<td>D₃₄</td>
</tr>
<tr>
<td>S₄</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Observed distance

Distance along tree

Goal:

\[ D_{12} \approx d_{12} = a + b + c \]
\[ D_{13} \approx d_{13} = a + d \]
\[ D_{14} \approx d_{14} = a + b + e \]
\[ D_{23} \approx d_{23} = d + b + c \]
\[ D_{24} \approx d_{24} = c + e \]
\[ D_{34} \approx d_{34} = d + b + e \]
Optimal Branch Lengths: Least Squares

- Fit between given tree and observed distances can be expressed as “sum of squared differences”:
  \[ Q = \sum_{j>i} (D_{ij} - d_{ij})^2 \]

- Find branch lengths that minimize \( Q \) - this is the optimal set of branch lengths for this tree.

**Goal:**
- \( D_{12} \approx d_{12} = a + b + c \)
- \( D_{13} \approx d_{13} = a + d \)
- \( D_{14} \approx d_{14} = a + b + e \)
- \( D_{23} \approx d_{23} = d + b + c \)
- \( D_{24} \approx d_{24} = c + e \)
- \( D_{34} \approx d_{34} = d + b + e \)
Least Squares Optimality Criterion

- Search through all (or many) tree topologies
- For each investigated tree, find best branch lengths using least squares criterion
- Among all investigated trees, the best tree is the one with the smallest sum of squared errors.
Exhaustive search impossible for large data sets

<table>
<thead>
<tr>
<th>No. taxa</th>
<th>No. trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>105</td>
</tr>
<tr>
<td>7</td>
<td>945</td>
</tr>
<tr>
<td>8</td>
<td>10,395</td>
</tr>
<tr>
<td>9</td>
<td>135,135</td>
</tr>
<tr>
<td>10</td>
<td>2,027,025</td>
</tr>
<tr>
<td>11</td>
<td>34,459,425</td>
</tr>
<tr>
<td>12</td>
<td>654,729,075</td>
</tr>
<tr>
<td>13</td>
<td>13,749,310,575</td>
</tr>
<tr>
<td>14</td>
<td>316,234,143,225</td>
</tr>
<tr>
<td>15</td>
<td>7,905,853,580,625</td>
</tr>
</tbody>
</table>
How many unrooted trees are there?

<table>
<thead>
<tr>
<th>$N_{\text{tips}}$</th>
<th>$N_{\text{trees}}$</th>
<th>$N_{\text{branches}} = N_{\text{derived trees}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>$2 \times 3 - 3 = 3$</td>
</tr>
<tr>
<td>4</td>
<td>$1 \times 3$</td>
<td>$2 \times 4 - 3 = 5$</td>
</tr>
<tr>
<td>5</td>
<td>$1 \times 3 \times 5$</td>
<td>$2 \times 5 - 3 = 7$</td>
</tr>
<tr>
<td>6</td>
<td>$1 \times 3 \times 5 \times 7$</td>
<td>$2 \times 6 - 3 = 9$</td>
</tr>
<tr>
<td>7</td>
<td>$1 \times 3 \times 5 \times 7 \times 9$</td>
<td>$2 \times 7 - 3 = 11$</td>
</tr>
<tr>
<td>8</td>
<td>$1 \times 3 \times 5 \times 7 \times 9 \times 11$</td>
<td>$2 \times 8 - 3 = 13$</td>
</tr>
<tr>
<td>9</td>
<td>$1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13$</td>
<td>...</td>
</tr>
</tbody>
</table>

$N_{\text{trees with n tips}} = \prod_{i=2}^{n-1} (2i - 3)$
Heuristic search

1. Construct initial tree; determine sum of squares

2. Construct set of “neighboring trees” by making small rearrangements of initial tree; determine sum of squares for each neighbor

3. If any of the neighboring trees are better than the initial tree, then select it/them and use as starting point for new round of rearrangements. (Possibly several neighbors are equally good)

4. Repeat steps 2+3 until you have found a tree that is better than all of its neighbors.

5. This tree is a “local optimum” (not necessarily a global optimum!)
Heuristic search: hill-climbing
Superimposed Substitutions

- Actual number of evolutionary events: 5
- Observed number of differences: 2
- Distance is (almost) always underestimated
- It is possible to estimate real distance from observed distance if one makes assumptions about how evolution has occurred (substitution frequencies etc).